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LETTER TO THE EDITOR

Non-topological Chern–Simons vortices in a scale-invariant tricritical Abelian Higgs model

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Abstract. It is shown that the scale-invariant tricritical $\lambda_1^2(\phi^*\phi)^3$-Abelian Higgs model with the Chern–Simons term has non-topological charged vortex solutions of finite energy, fractional angular momentum and anomalous scale dimension. For a specific choice of the coupling λ_1, first-order Bogomolny-type equations are obtained and it is shown that unlike the second-order equations, these first-order equations do not admit charged vortex solutions of finite energy.

In recent years there has been considerable interest in the charged vortex solutions of the Abelian Higgs model with Chern–Simons (CS) term in 2+1 dimensions [1]. By now, topological as well as non-topological charged vortex solutions have been obtained in the case where the Higgs potential corresponds to a first- [2] as well as second-order [3] phase transition. Further, for a specific choice of the Higgs potential (which corresponds to the first-order transition point) and when the gauge field kinetic energy is absent, the vortex satisfies a set of self-dual or Bogomolny-type first-order coupled equations [4]. It turns out that with this specific Higgs potential the bosonic Lagrangian forms a part of the $N=2$ supersymmetric theory [5].

The purpose of this letter is to extend this discussion to a model which is scale invariant and furthermore to show that the corresponding Higgs potential $(\lambda_1^2(\phi^*\phi)^3)$ corresponds to the tricritical behaviour. It may be pointed out here that the tricritical models are of considerable importance in statistical mechanics [6]. There has also been some discussion about them from the field theory point of view [7]. It is shown that this model admits non-topological charged vortex solutions of finite energy, charge and flux, fractional angular momentum and anomalous scale dimension. These are unusual vortices in the sense that both the magnetic field and the Higgs field have only power-law fall-off at large distance. For a specific choice of the Higgs coupling λ_1, the bosonic Lagrangian forms a part of the $N=2$ supersymmetric theory. Not surprisingly, as in the first-order transition case [4], one finds that at this value of the coupling, one has self-dual or Bogomolny-type coupled first-order equations. However, unlike in that case, it turns out that the two first-order equations are incompatible with each other and no charged vortex solution exists for these first-order equations (even though the solution exists to the coupled second-order equations).

The Lagrangian density for the tricritical Abelian Higgs model with the pure CS term in 2+1 dimensions is given by

$$\mathscr{L}=\tfrac{1}{2}(\partial_\mu - \mathrm{i}eA_\mu)\phi^*(\partial^\mu + \mathrm{i}eA^\mu)\phi - \tfrac{1}{8}\lambda_1^2(\phi^*\phi)^3 + \tfrac{1}{4}K\varepsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda . \qquad (1)$$

Note that in the presence of the gauge field kinetic energy, the mass dimension of the gauge potential A_μ is $\frac{1}{2}$ and hence the mass dimensions of e and K are $\frac{1}{2}$ and 1 respectively. However, in the absence of the gauge field kinetic energy, the mass

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dimension of A_μ can also be taken as 1 so that e, K and λ_1 are all dimensionless parameters and hence the model given by (1) is scale invariant (the dimension of ϕ is $\frac{1}{2}$). Unless stated otherwise, this is the choice that I shall consider in this paper. On using the same ansatz as in [1] i.e. ($\rho \geqslant 0, 0 \leqslant \theta \leqslant 2\pi$)

$$A(\boldsymbol{\rho}, t) = -e_\theta \frac{A(\rho)}{\rho} \qquad A_0(\boldsymbol{\rho}, t) = A_0(\rho)$$
$$\phi(\boldsymbol{\rho}, t) = e^{in\theta} F(\rho) \qquad n = 0, \pm 1, \pm 2, \ldots . \tag{2}$$

The following field equations are then obtained

$$-e(n+eA)F^2 = K\rho A_0'(\rho) \tag{3a}$$

$$-e^2 A_0 F^2 = \frac{K}{\rho} A'(\rho) \tag{3b}$$

$$F''(\rho) + \frac{1}{\rho} F'(\rho) + e^2 A_0^2 F - (n+eA)^2 \frac{F}{\rho^2} - \frac{3\lambda_1^2}{4} F^5 = 0. \tag{3c}$$

It would appear from this that the field equations depend on three dimensionless parameters e, K and λ_1. However this is not so and two of them can be scaled away by defining new variables

$$g = n + eA \qquad h = r_0 \frac{A_0}{e} \qquad f = \sqrt{\frac{r_0}{K}} F \qquad r = \frac{e^2}{r_0} \rho \qquad \lambda = \frac{K}{e^2} \lambda_1 \tag{4}$$

where r is a dimensionless variable ($0 \leqslant r \leqslant \infty$) and r_0 is an arbitrary parameter with the dimension of length. In terms of these new variables, the field equations (3) take the form

$$-gf^2 = rh'(r) \tag{5a}$$

$$-hf^2 = \frac{1}{r} g'(r) \tag{5b}$$

$$f''(r) + \frac{1}{r} f'(r) + h^2 f - \frac{g^2 f}{r^2} - \frac{3}{4} \lambda^2 f^5 = 0. \tag{5c}$$

The corresponding field energy can be shown to be

$$E = \frac{\pi K}{r_0} \int_0^\infty r\, dr \left[\frac{\lambda^2}{4} f^6 + \left(\frac{df}{dr} \right)^2 + h^2 f^2 + \frac{g^2}{r^2} f^2 \right]. \tag{6}$$

I have not been able to obtain an analytical solution to these field equations. However for $r \to 0$ and $r \to \infty$, I can show that the field equations (5) admit non-topological charged vortex solutions of finite energy. From (6) it is clear that asymptotically $f \to 0$. Further, since there is no spontaneous symmetry breaking, no gauge field kinetic energy and no f^2 term in (1) hence the gauge as well as the Higgs fields are massless and hence must have a power-law fall-off as $r \to \infty$. It is easily seen that (5) admit the following solution

$$f(r) = \frac{a_\infty}{r^\alpha} + \frac{a_1}{r^{5\alpha - 2}} + \ldots \tag{7a}$$

$$g(r) = -\alpha + \frac{a_\infty^4}{4(2\alpha - 1) r^{4\alpha - 2}} + \ldots \tag{7b}$$

$$h(r) = \frac{a_\infty^2}{2r^{2\alpha}} + \frac{a_2}{r^{6\alpha - 2}} + \ldots \tag{7c}$$

where

$$a_1=\frac{[3\lambda^2(2\alpha-1)+1-4\alpha]}{16(3\alpha-1)(2\alpha-1)^2}a_\infty^5$$
$$a_2=\frac{[3\lambda^2\alpha(2\alpha-1)+8\alpha^2-9\alpha+2]}{16(3\alpha-1)^2(2\alpha-1)^2}a_\infty^6. \tag{8}$$

Here a_∞ and α are two arbitrary constants. Whereas finite energy considerations restrict α to $\alpha>\frac{1}{3}$, the fact that the solution (7) must be a power series solution in decreasing powers of r (i.e. $5\alpha-2>\alpha$) implies that $\alpha>\frac{1}{2}$. Hence, as $r\to\infty$, the Higgs field ϕ, the electric field $E_\rho(=\mathrm{d}A_0/\mathrm{d}r)$ and the magnetic field $B(\equiv-(1/\rho)(\mathrm{d}A/\mathrm{d}\rho)$ scale with anomalous scale dimension α, $2\alpha+1$ and 4α respectively.

It is tempting to enquire if there are solutions to (7) with only logarithmic scaling violation. The answer to the question is no, unless $\lambda=1$. In particular one can show that at $\lambda=1$ and as $r\to\infty$, equations (7) also admit the solution

$$f(r)=\frac{\pm1}{r^{1/2}(\log r)^{1/2}}+\frac{a_\infty}{r^{1/2}(\log r)^{3/2}}+\ldots \tag{9a}$$

$$g(r)=-\tfrac{1}{2}-\frac{1}{2(\log r)}\mp\frac{a_\infty}{(\log r)^2}+\ldots \tag{9b}$$

$$h(r)=-\frac{1}{2r(\log r)}\mp\frac{a_\infty}{r(\log r)^2}+\ldots. \tag{9c}$$

For $r\to0$ it is easily seen that the field equations (5) admit the following solution (without loss of generality we choose $n>0$; a similar solution is also valid for $n<0$)

$$f(r)=a_0r^n+a_1r^{n+2}+\ldots \tag{10a}$$

$$g(r)=n+a_2r^{2n+2}+\ldots \tag{10b}$$

$$h(r)=b_0-\frac{a_0^2}{2}r^{2n}+\ldots \tag{10c}$$

with

$$a_1=\frac{-b_0^2a_0}{4(n+1)}\qquad a_2=-\frac{b_0a_0^2}{2(n+1)}. \tag{11}$$

It is worth pointing out here that unlike the topological case the non-topological charged vortex solution exists even when $n=0$. In this case for $r\to\infty$ the solution is again given by (7) while for $r\to0$ it is given by

$$f(r)=a_0+b_1r^2+\ldots \tag{12a}$$

$$g(r)=-\frac{b_0a_0^2}{2}r^2+b_2r^4+\ldots \tag{12b}$$

$$h(r)=b_0+\frac{b_0}{4}a_0^4r^2+\ldots \tag{12c}$$

where

$$b_1=\left(\frac{3\lambda^2}{4}a_0^4-b_0^2\right)\frac{a_0}{4}\qquad b_2=\frac{a_0^2b_0}{8}\left(b_0^2-\frac{a_0^4}{2}-\frac{3}{4}a_0^4\lambda^2\right). \tag{12d}$$

A few remarks are in order at this stage.

(i) These non-topological vortices have non-zero flux Φ, charge Q, and angular momentum J which are given by [1] ($n=0,+1,+2,\ldots$)

$$\Phi=\int \mathrm{d}^2\rho\, B(\rho)=\frac{2\pi}{e}(n+\alpha) \tag{13a}$$

$$Q=K\Phi=\frac{2\pi K}{e}(n+\alpha) \tag{13b}$$

$$J=-\frac{\pi K}{e^2}(n^2-\alpha^2)=\frac{Q\Phi}{4\pi}-\frac{n}{e}Q. \tag{13c}$$

Note that unlike the usual vortices in 2+1 dimensions, in this scale-invariant model, both Φ and Q (and of course also J) are dimensionless. Further, as in order CS vortex cases, J is, in general, fractional so that these objects are anyons [8].

(ii) The magnetic moment μ_z of these vortices can be computed by using field equations (5). We find

$$\mu_z\equiv\tfrac{1}{2}\int \mathrm{d}^2\rho(\bar{\rho}'\times\bar{\gamma}')_z=\frac{2\pi K}{e^3}\int_0^\infty r\,\mathrm{d}r\, h(r). \tag{14}$$

(iii) Physically, the vortex solutions for $n=0$ and $n\neq 0$ are very different. For $n\neq 0$ the magnetic field is peculiar in the sense that it vanishes both at the core of the vortex and also as $r\to\infty$ and is a maximum in between. Note also that the Higgs field has a similar profile. On the other hand, for $n=0$ we have the unusual situation of both magnetic and Higgs fields being non-zero and finite at $r=0$ and both of them have power-law fall-off as $r\to\infty$.

(iv) The entire discussion is obviously also valid in the case of $\lambda_1=0$, i.e. when there is no Higgs potential. Such a scale-invariant model has recently been quantized and its anyonic features have been brought out quite clearly [9].

Finally let us discuss the question of the self-dual or Bogomolny-type equations. If one carefully goes through the arguments of Lee *et al* [5] then it is clear that in the case $\lambda_1=e^2/K$ (i.e. $\lambda=1$) then the bosonic Lagrangian (1) forms a part of the $N=2$ supersymmetric theory. In this case, on using the constraint (5b) the vortex energy energy as given by (6) can be rewritten as

$$E=\pi K\int_0^\infty r\,\mathrm{d}r\left[\left(\frac{\mathrm{d}f}{\mathrm{d}r}\mp\frac{gf}{r}\right)^2+\left(\frac{1}{r}f^{-1}\frac{\mathrm{d}g}{\mathrm{d}r}\mp\frac{f^3}{2}\right)^2\pm\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(gf^2)\right]. \tag{15}$$

We thus have a (trivial) lower bound ($E\geqslant 0$) on the energy which is saturated by fields obeying the self-duality or Bogomolny equations

$$\frac{\mathrm{d}f}{\mathrm{d}r}=\pm\frac{gf}{r} \tag{16a}$$

$$\frac{1}{r}\frac{\mathrm{d}g}{\mathrm{d}r}=\pm\tfrac{1}{2}f^4 \tag{16b}$$

where the upper (lower) sign corresponds to positive (negative) values of n. However, it turns out that these first-order equations are mutually inconsistent. For example using the power series ansatz similar to those given by (7) and (10) one finds that for $n>0$ ($n<0$) whereas (16a) implies a positive (negative) flux, just the opposite conclusion follows from (16b). Thus these first-order equations do not admit charged vortex solutions. Of course this is quite understandable since the trivial solution $h = g = f = 0$ has zero energy, hence any non-trivial non-topological solution must necessarily have positive energy.

Recently we have also obtained non-topological neutral as well as charged vortices with power-law fall-off in systems exhibiting first- and second-order transitions, the details of which will be published elsewhere [10].

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